TEEHR Metrics

RTI International

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Mean Error:

$$Mean\ Error = \frac{\sum(sec - prim)}{count} \tag{1}$$

Relative Bias:

$$Relative\ Bias = \frac{\sum(sec - prim)}{\sum(prim)} \tag{2}$$

Multiplicative Bias:

$$Mult.\ Bias = \frac{ave(sec)}{ave(prim)} \tag{3}$$

Mean Square Error:

$$MSE = \frac{\sum(sec - prim)^2}{count} \tag{4}$$

Root Mean Square Error:

$$RMSE = \sqrt{\frac{\sum(sec - prim)^2}{count}} \tag{5}$$

Mean Absolute Error:

$$MAE = \frac{\sum|sec - prim|}{count} \tag{6}$$

Mean Absolute Relative Error:

$$Relative\ MAE = \frac{\sum|sec - prim|}{\sum(prime)} \tag{7}$$

Pearson Correlation Coefficient:

$$r = r(sec, prim) \tag{8}$$

Coefficient of Determination:

$$r^2 = r(sec, prim)^2 \tag{9}$$

Nash-Sutcliffe Efficiency:

$$NSE = 1 - \frac{\sum(prim - sec)^2}{\sum(prim - ave(prim)^2)} \tag{10}$$

Normalized Nash-Sutcliffe Efficiency:

$$NNSE = \frac{1}{(2 - NSE)} \tag{11}$$

Kling Gupta Efficiency - original:

$$KGE = 1 - \sqrt{((corr(sec, prim)) - 1)^2 + ((\frac{stddev(sec)}{stddev(prim)} - 1)^2) + (\frac{avg(sec)}{avg(sec)/avg(prim)} - 1)^2} \quad (12)$$

Kling Gupta Efficiency - modified 1 (2012):

$$KGE' = 1 - \sqrt{((corr(sec, prim)) - 1)^2 + ((\frac{\frac{stddev(sec)}{ave(sec)}}{\frac{stddev(prim)}{ave(prim)}} - 1)^2) + (\frac{ave(sec)}{avg(sec)/avg(prim)} - 1)^2} \quad (13)$$

Kling Gupta Efficiency - modified 2 (2021):

$$KGE" = 1 - \sqrt{((corr(sec, prim)) - 1)^2 + ((\frac{stddev(sec)}{stddev(prim)} - 1)^2) + (\frac{(avg(sec) - avg(prim))^2}{stddev(prim)^2}} \quad (14)$$

Nash-Sutcliffe Efficiency of Log Flows:

$$NSE(log) = 1 - \frac{\sum(log(prim) - log(sec))^2}{\sum(log(prim) - ave(log(prim)))^2} \quad (15)$$

Annual Peak Flow Relative Bias:

$$Ann\ PF\ Bias = \frac{\sum(sec\ ann.\ peak - prim\ ann.\ peak)}{\sum(prim\ ann.\ peak)} \quad (16)$$

Spearman Rank Correlation Coefficient:

$$r_s = 1 - \frac{6 * \sum |prim_{rank} - sec_{rank}|^2}{count(count^2 - 1)} \quad (17)$$

Flow Duration Curve Slope Error:

$$Slope\ FDC\ Error = \frac{sec_{q66} - sec_{q33}}{33} - \frac{prim_{q66} - prim_{q33}}{33} \quad (18)$$

Event Peak Flow Relative Bias:

$$Peak\ Bias = \frac{\sum(sec\ peak - prim\ peak)}{\sum(prim\ peak)} \quad (19)$$

Event Peak Flow Timing Error:

$$Peak\ Time\ Error = \frac{\sum(sec\ peak\ time - prim\ peak\ time)}{count} \quad (20)$$

Baseflow Index Error:

$$BFI\ Error = \frac{\frac{ave(sec_{baseflow})}{ave(sec)} - \frac{ave(prim_{baseflow})}{ave(prim)}}{\frac{ave(prim_{baseflow})}{ave(prim)}} \quad (21)$$

Rising Limb Density Error:

$$RLD\ Error = \frac{count(sec\ rising\ limb\ events)}{count(sec\ rising\ limb\ timesteps)} - \frac{count(prim\ rising\ limb\ events)}{count(rising\ limb\ timesteps)} \tag{22}$$

Mean Square Error Skill Score (generalized reference):

$$MSESS = 1 - \frac{\sum(prim - sec)^2}{\sum(prim - reference)^2} \tag{23}$$

Runoff Ratio Error:

$$RR\ Error = abs \left\| \frac{ave(secvolume)}{ave(precipvolume)} - \frac{ave(primvolume)}{ave(precipvolume)} \right\| \tag{24}$$

False Alarm Ratio:

$$FAR = \frac{n_{FP}}{n_{TP} + n_{FP}} \tag{25}$$

Probability of Detection:

$$POD = \frac{n_{TP}}{n_{TP} + n_{FN}} \tag{26}$$

Probability of False Detection:

$$POFD = \frac{n_{FP}}{n_{TN} + n_{FP}} \tag{27}$$

Critical Success Index (Threat Score):

$$CSI = \frac{n_{TP}}{n_{TP} + n_{FN} + n_{FP}} \tag{28}$$

Brier Score:

$$BS = \frac{\sum(sec\ ensemble\ prob - prim\ outcome)^2}{n} \tag{29}$$

Brier Skill Score:

$$BSS = 1 - \frac{BS}{BS_{ref}} \tag{30}$$

Continuous Ranked Probability Skill Score:

$$CRPSS = 1 - \frac{CRPS}{CRPS_{ref}} \tag{31}$$